Building upon renewed interest in mathematics education in the epistemology of mathematics, this study investigated college students’ attempts to reason mathematically. The students in the study had not received prior training in proof writing and making. The central research question was: What are the students’ modes of reasoning and how do the observed modes relate to the rules of mathematical discourse? To this end, I examined students exploring calculus problems in which they posed conjectures, communicated their insights, and ascertained the truthfulness of their claims with different degrees of confidence.

The study was a naturalistic teaching experiment in which cognitive, metacognitive, and social aspects of students’ proofs were examined. There were eight key participants: four undergraduate calculus students enrolled in a Calculus II course, two recent high school graduates who had completed an Advanced Placement calculus course, and two graduate mathematics education students who had recently completed their undergraduate work. The data were mainly collected through problem-solving interviews with the key participants. To establish a context of the research setting, classroom observations and informal interviews were also conducted. Through the constant comparison method, the categories and their properties were inductively developed from the data to generate a grounded theory of students’ ways of knowing within mathematics.

The findings revealed that the students used plausible reasoning as a means of inference making. Their inferences, although not formally represented, could sometimes be extended to more formal contexts. Often, however, the students’ attempts to operate within the constraints of mathematical discourse were immature and futile. The main sources of students’ difficulties were: (a) lacking oral, aural, and written fluency in mathematics, (b) avoiding mathematical
vocabulary and explicitness, (c) naively using metaphors as substitutes for definitions, and (d) participating in classroom practices that do not promote proving behavior. Naive empiricism was also used as a source of conviction. Its origin stems not only from beliefs but also from the lack of intrinsic interest in finding a coherent explanation to account for a mathematical result. Sometimes, pragmatic explanation was used out of desperation and social tensions caused by the problem situation.