Mathematical Knowledge for Teaching Teachers: The Case of Multiplication and Division of Fractions

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Dissertation Abstract

This study attempts to answer the question, *What is the mathematical knowledge required by teachers of elementary mathematics content courses in the area of multiplication and division of fractions?* Beginning in the mid-1980s, when Shulman (1986) introduced the idea of pedagogical content knowledge, researchers have been looking at the knowledge needed to teach in a variety of different content areas. One area that has garnered much of the research is that of mathematics. Researchers have developed frameworks for what they call mathematical knowledge for teaching, but there has been little work done looking at the knowledge requirements for teachers of teachers. This study attempts to fill this gap by determining some aspects of a framework for the mathematical knowledge required to teach prospective elementary teachers multiplication and division of fractions.

In order to determine aspects of a framework for mathematics teacher educator knowledge in relation to multiplication and division of fractions, I interviewed, observed, and audiotaped three experienced teacher educators in different educational settings to determine the mathematical work of teaching prospective teachers fraction multiplication and division. My analysis focused on three of major tasks that came out of the work: introducing fraction multiplication, helping students make sense of fraction division, and assessing student
understanding. Each of these tasks played a major role in the work of the teacher educators, and the knowledge required to perform these tasks was evident in varying degrees in each teacher educator.

After analyzing the three mathematical tasks and the knowledge required by them, I was able to determine some components of a framework for the mathematical knowledge needed for teaching teachers multiplication and division of fractions. These aspects include: understanding multiple representations of fraction multiplication and division and how these representations relate to each other, to whole number ideas, and to the algorithms, deciding which aspects of the topics will help prospective teachers make the connections that they will need in order to teach these topics, especially since time often plays a factor in what gets taught in mathematics content classes for prospective teachers, setting specific goals of exactly what one wants one’s students to know, rather than having a general goal of wanting prospective teachers to develop conceptual understanding of a topic, and being able to design and use assessments effectively to help decide if one is achieving one’s goals.

While each of the aspects described above are components of a framework for the mathematical knowledge needed by teacher educators, the three teacher educators in my study all lacked or were unable to demonstrate some of the knowledge components that would have helped them to meet their goals, despite having a wealth of experience teaching and designing mathematics content courses for prospective elementary teachers. One possible reason for this is that each of the teacher educators in my study were basically alone in their departments, without opportunities to collaborate or discuss these ideas with anyone else. These results suggest a need for better professional development for teacher educators in the field of mathematics education.