Comparison of Four Instructional Approaches and Mathematics
Background on Students’ Conceptions of Limits

William J. Hardin
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Committee:  Prof. Joanna O. Masingila, Chair
Prof. Theodore Cox
Prof. Helen M. Doerr
Prof. Emily Robertson

Dissertation Abstract

Limits are central to calculus. They are what separate analysis from algebra. Unfortunately, many students find limits to be a difficult and confusing topic. In this dissertation, I attempt to see how different ways of teaching the concept of limits affect students’ conceptions of limits as well as their computational proficiency.

To accomplish my goals, I taught four different recitation sections, each with a different instructional approach. The treatments were conducted for a total of three weeks. The instructional approaches were: paper-and-pencil computational, graphics calculator-based, paper-and-pencil conceptual, and computer-based. All the groups except the paper-and-pencil computational group emphasized conceptual knowledge first. I found that emphasizing conceptual knowledge (except for the graphics calculator group) resulted in students that were stronger in conceptual knowledge, while emphasizing computation yielded students stronger in computation. A notable exception was the paper-and-pencil conceptual group, that was at the top of the groups on scores on both conceptual and procedural problems. In the instructional activities, the students in the paper-and-pencil conceptual group were asked to compute parameters like slopes of tangents themselves. This may give some indication that forcing the students to process the information may cause the connections to become more explicit.

During the investigation, I developed software to help students visualize the concept of limits. This software let students dynamically control important parameters. In this way, the software acted like a virtual manipulative. I also enhanced a classification model for students’ conceptions that was developed by Williams (1991). Additionally, I attempted to classify errors based on a classification model by Movshovitz-Hadar, Zaslavsky and Inbar without too much success.
Finally, I developed a set of activities for each of the instructional approaches. The activities were similar between the pencil-and-paper conceptual and the computer-based sections but different for the calculator and the paper-and-pencil computational groups. These activities rely on the development of exposing, discrepant and resolving events to bring the notion of limits to light. The activities were designed so that the computational group did not have discussions; instead they practiced computational problems. However, in the activities the conceptual groups were asked to make conjectures and were designed as the foundation of class discussions.