High School Mathematics Teachers’ Understandings of the Purposes of Mathematical Proof

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Dissertation Abstract

This qualitative study reports on high school mathematics teachers’ understandings of mathematical proof and its purposes, as both persons knowledgeable in mathematics and as teachers of mathematics and has implications for the professional development of high school mathematics teachers.

Four reasons participants gave for teaching proof were (1) proof is integral to mathematics; (2) verification and discovery of facts; (3) development of students’ thinking; and (4) showing that mathematics is a human endeavor. The development of student thinking was a frequently cited reason for teaching proof. Specifically, proofs were perceived to (1) foster mathematical understanding, and (2) help develop students’ transferable thinking skills. Proofs were said to help develop students’ mathematical thinking skills in such ways as solidifying students’ mathematical knowledge, providing deeper insight into mathematics, or helping students to think mathematically. But more frequently, the teachers spoke of helping students to develop transferable thinking skills. These skills were perceived to be useful outside of mathematics classrooms and were perceived to have life-long benefits to the students.

Some teachers expressed reservations regarding the certainty of mathematical knowledge. The most common of these reservations was that it is possible for a theorem to be validated in error by a succession of mathematicians, each of whom had failed to find a vital flaw. But a reservation held by at least two of the teachers was that even in mathematics, there are no absolute truths.

There were three ways in which a proof could be unclear to a reader that were identified by the participants. First, the reading of a proof might require advanced mathematical knowledge inaccessible to the reader; the logical structure of the proof might elude the reader; and finally,
even a proof that is otherwise accessible might require too much work or effort on the part of the 
reader. Finally, teachers with less than ten years of experience tended to be more interested in 
precise language and explicit logic whereas teachers with more than twenty years of experience 
tended to be more open to alternative forms of argumentation.